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PROPAGATION OF LOVE WAVES IN PRESTRESSED ORTHOTROPIC LAYER COATED OVER A PRESTRESSED ORTHOTROPIC SEMI-INFINITE SPACE WITH IRREGULAR INTERFACE

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Abstract: This paper contains the dispersion equation of Love waves propagating in an initially-stressed orthotropic layer coated over an initially-stressed orthotropic semi-infinite space. Rectangular irregularity of height H and width b at the interface is considered in this paper. The surface of the layer is supposed to be traction-free. To find the numerical results, semi-infinite space is assumed to be of topaz material and the layer is assumed to be of olivine material. Based on the dispersion equation, numerical values of dimensionless velocity against dimensionless layer thickness are calculated. The comparison of the velocity curves for different values of initial stress has been examined graphically. Graphs for one and two modes of velocity are drawn. The effect of irregularity on the velocity of Love waves has been shown graphically. It is shown that the second mode of velocity shows more variation than the first mode of velocity. MATLAB is used to plot graphs. It is demonstrated that initial-stress and irregular interfaces significantly affect the velocity of Love waves.

Keywords and Phrases: Orthotropic, irregularity, initial stress, Love waves, velocity.

2020 Mathematics Subject Classification: 74J15, 86A15.

1. Introduction

Surface waves are the most destructive type of elastic wave and travel at low velocity. Love waves are surface waves in which particles vibrate in a horizontal direction. The information collected by waves propagating in various composite media can be used to study the earth's interior. The stress that exists in the absence of external body forces in a body is known as "initial stress." Physical considerations like cold working, gravity changes, and the gradual creep process cause initial stress in the material. Furthermore, the earth's surface is an elastic medium that is subjected to significant initial stresses. As a result, it's critical to examine the impact of initial stress on wave propagation within the body or across the earth's surface. The propagation of Love waves in prestressed inhomogeneous layers due to point sources is discussed by Chattopadhyay and Kar [4]. Vishwakarma et al. [16] derived a dispersion relation for Love Waves propagating in an orthotropic layer which is perfectly bonded with a layered anisotropic-porous material in semi-infinite space. The surface waves in a prestressed orthotropic layer lying over a semi-infinite space have been examined by many researchers like Rani and Madan [13], Saha et al. [14], Madan et al. [12], and more.

The effect of initial stress on the Love wave velocity in a layer in welded contact with semi-infinite space is discussed by Gupta et al. [7] and more. Love waves in a porous layer coated over a semi-infinite space are investigated by Wang [17] and more. Due to faults or thermal mismatch, some defects may occur at the time of manufacturing some materials. These defects give an imperfect interface. Problems that explore the effect of elastic constants and material gradients on the velocity of surface waves are very helpful. Madan and Kumari [11], Chaudhary et al. [5], Kumari and Madan [10] and others investigated the effect of an imperfect interface on the propagation of waves. Goyal and Sahu [6] examined Love wave propagation in a piezo layered substrate with an imperfect interface between is considered.

A drop in velocity is observed when the wave number rises. Jin et al. [9] examined the effect of imperfect bonding of the layer with the substrate on the propagation of Love waves. Hua et al. [8] analyse the propagation of Love waves in layered graded composites in which imperfect bonding is considered. The dispersion equation for Love waves propagating in porous layered semi-infinite space with an irregular interface is derived by Chattopadhyay and De [3], Chattaraj et al. [2], and more. Singh et al. [15] discussed Love wave propagation in layered-isotropic semi-infinite space. Irregular boundaries are considered in this paper.
The thermal or mechanical properties of orthotropic materials are unique. These facts inspire us to examine Love waves in an orthotropic medium. In the above papers, Love wave propagation in a prestressed orthotropic layer over a prestressed

orthotropic semi-infinite space is not discussed. In this work, the Love waves in a prestressed orthotropic layer lying over an orthotropic semi-infinite space with irregularity are discussed. The effect of irregularity and initial stress on velocity is examined graphically.

2. Basic theory and Problem Formulation

The equation of motion in an initially-stressed orthotropic medium is given by (Biot [1])

$$Q_1 \frac{\partial^2 u_2}{\partial z^2} + \left(Q_3 - \frac{P}{2} \right) \frac{\partial^2 u_2}{\partial x^2} = \rho \frac{1}{\bar{c}_2^2} \frac{\partial^2 u_2}{\partial t^2} \tag{1}$$

where $P = -S_{11}$, ρ is density, Q_1, Q_3 are the incremental elastic coefficients and S_{11} is initial stress component in x direction.

Consider an initially-stressed orthotropic semi-infinite space $z \geq 0$ coated by an initially-stressed orthotropic layer $-h \leq z \leq 0$ of width h with irregular interface (Fig. 1). A rectangular irregular model of height H and width b at the interface is considered. Quantities associated with semi-infinite space and layers have similar notation but are distinguished by bars for the layer. in this paper, the antiplane-strain problem is considered in such a way that $u_2 = u_2\,(x, z, t)$, $\bar{u}_2 = \bar{u}_2\,(x, z, t)$ where $u_2, \bar{u}_2$ are the displacement components, t is time.

$$F(x) = \begin{cases} 0 & |x| > b \\ 2b & |x| \leq b \end{cases} \tag{2}$$

$$\epsilon = \frac{H}{2b} << 1 \tag{3}$$

Let $$P = -S_{11}, \bar{P} = -\bar{S}_{11}, P_H = \frac{P}{Q_3}, P_L = \frac{\bar{P}}{\bar{Q}_3}, e_1 = \frac{Q_3 - \frac{P}{2}}{Q_3}, e_2 = \frac{Q_1}{Q_3},$$

$$r_u = \frac{\bar{Q}_3}{Q_3}, c_2 = \sqrt{\frac{Q_3}{\rho}}, \bar{c}_2 = \sqrt{\frac{\bar{Q}_3}{\bar{\rho}}}, r_v = \frac{c_2}{\bar{c}_2}, c_T = \frac{kq}{k_T}, C = \frac{c^2}{\bar{c}_2^2}, s_{23} = \frac{\sigma_{23}}{Q_3},$$

$$\bar{s}_{23} = \frac{\bar{\sigma}_{23}}{\bar{Q}_3}, v = \frac{u_2}{h}, v = \frac{\bar{u}_2}{h} \tag{4}$$

The components of Stress in initially-stressed orthotropic medium (in dimensionless form) are

$$\bar{s}_{23} = \bar{e}_2 \frac{\partial \bar{v}}{\partial z} \tag{5}$$

The Equation of motion (in dimensionless form) is,

$$\bar{e}_1\frac{\partial^2 v}{\partial x^2}+\bar{e}_2\frac{\partial^2 v}{\partial z^2}=\frac{1}{\bar{c}_2^2}\frac{\partial^2 v}{\partial t^2} \tag{6}$$

The displacement components of the Love wave in the layer satisfying Equation (6), are

$$\bar{v}=\bar{V}\left(x_3\right)\exp ik(x-ct),\quad \text{where } x_3=kz \text{ and} \tag{7}$$

$$\bar{V}=G_1\cosh\left(mx_3\right)+G_2\sinh\left(mx_3\right),\quad \text{where}\quad m^2=\frac{\left(\bar{e}_1-C\right)}{\bar{e}_2} \tag{8}$$

Using Equations (7) and (8) in Equation (5) leads to

$$\bar{s}_{23}=k\bar{S}\left(x_3\right)\exp ik(x-ct), \tag{9}$$

$$\text{where}\quad \bar{S}\left(x_3\right)=m\bar{e}_2\left(G_1\sinh\left(mx_3\right)+G_2\cosh\left(mx_3\right)\right) \tag{10}$$

In the semi-infinite space $z>0$, displacement of Love waves is given by,

$$v=V\left(x_3\right)\exp ik(x-ct),\quad \text{where}\quad V\left(x_3\right)=G\exp\left(-gx_3\right) \tag{11}$$

$$g^2=\frac{\left(\bar{e}_1-\frac{c^2}{r_v^2}\right)}{\bar{e}_2} \tag{12}$$

Introducing Equations (11) and (12) into Equation (5) without bars lead to

$$s_{23}=kS\left(x_3\right)\exp ik(x-ct)\quad \text{in which}\quad S\left(x_3\right)=-e_2gG\exp\left(-gx_3\right) \tag{13}$$

3. Secular Equation for Irregular Interface

Boundary Conditions:
As the surface $z=-h$ is considered free, therefore

$$\bar{s}_{23}=0\quad \text{at}\quad z=-h. \tag{14}$$

Let the contact of layer and semi-infinite be welded, thus

$$v=\bar{v},\quad \bar{s}_{23}=s_{23}\quad at\quad z=\epsilon F\left(x\right). \tag{15}$$

Using Equations (7) - (13) in Equations (14) and (15) leads to

$$-G_1\sinh\left(me\right)+G_2\cosh\left(me\right)=0 \tag{16}$$

$$G_1 + G_2 mk\epsilon F - G\exp(-gk\epsilon F) = 0 \tag{17}$$

$$G_1 m^2 k\epsilon F + m\bar{e}_2 G_2 + ge_2(1 - gk\epsilon FG) = 0 \tag{18}$$

For the non-trivial solution of Equations (16)-(18), we have

$$(ge_2 \tanh(mkF) + m\bar{e}_2)\tanh(me) + ge_2 + m\bar{e}_2 \tanh(mkF) = 0 \tag{19}$$

This equation is the dispersion equation for Love waves propagating in prestressed orthotropic layer coated over a prestressed orthotropic semi-infinite space with irregular interface.

4. Numerical Results and Discussion

For numerical computation, we have used the olivine rock (magnesium iron silicate) for the orthotropic elastic layer (representing lithosphere) and topaz for the orthotropic semi-infinite space (representing asthenosphere). The parameters are considered as: $\rho = 3.55 \times (10)^{(11)} g/cm^3, \bar{\rho} = 3.05 \times (10)^{(11)} g/cm^3, Q_1 = 10.8 \times (10)^{(11)} dyne/cm^2, Q_3 = 12.55 \times (10)^{(11)} dyne/cm^2, \bar{Q_1} = 3.25 \times (10)^{(11)} dyne/cm^2$, $\bar{Q_3} = 4.55 \times (10)^{(11)} dyne/cm^2$, and dimensionless parameters are $e_1 = 1 - P_H/2, \bar{e}_1 = 1 - P_L/2, e_2 = 0.83, \bar{e}_2 = 0.71, r_u = 0.35, r_v = 1.57$.

In Fig. 2, the impact of initial stress$P_H(0.1, 0.3)$ and$P_L(0.1356, 0.3756)$,on phase velocity is shown. It is concluded that as initial stress increases, velocity of Love wave decreases. Lowest velocity of wave is observed at dimensionless layer thickness $e = 3.5$. In Fig. 3, the effect of $P_H(0.1, 0.4, 0.5)$ and $P_L(0.1356, 0.4856, 0.6)$, on phase velocity of two modes is shown. A significant effect of initial stress on the second mode of Love wave velocity is observed as compared to the first mode. Fig. 4 shows the effect of $P_H(0.1, 0.2, 0.4)$ and $P_L(0.1356, 0.2612, 0.4856)$, on phase velocity of two modes. The velocity decreases as layer thickness increases in Figs 2, 3, 4. A significant effect of irregularity and initial stress is observed on Love wave velocity.

Figures shows that:

1. The Love waves velocity decreases as dimensionless layer thickness increases.
2. Love waves velocity decreases as values of initial stress increases.
3. The velocity of second mode shows more variation against dimensionless layer thickness as compared to first mode.
4. Initial stress strongly affects the velocity of Love waves.

5. Conclusion

The effects of initial-stress and irregularity on the Love wave in the orthotropic elastic layer coated over orthotropic semi-infinite space are being investigated. As the initial stress value rises, a drop in Love wave velocity is observed. In comparison to the first mode, the velocity of the second mode has a greater impact of the initial stress. A significant effect of initial stress and irregularity is observed. This research has applications in a variety of industries, including civil engineering, geology, and others.

6. Figures

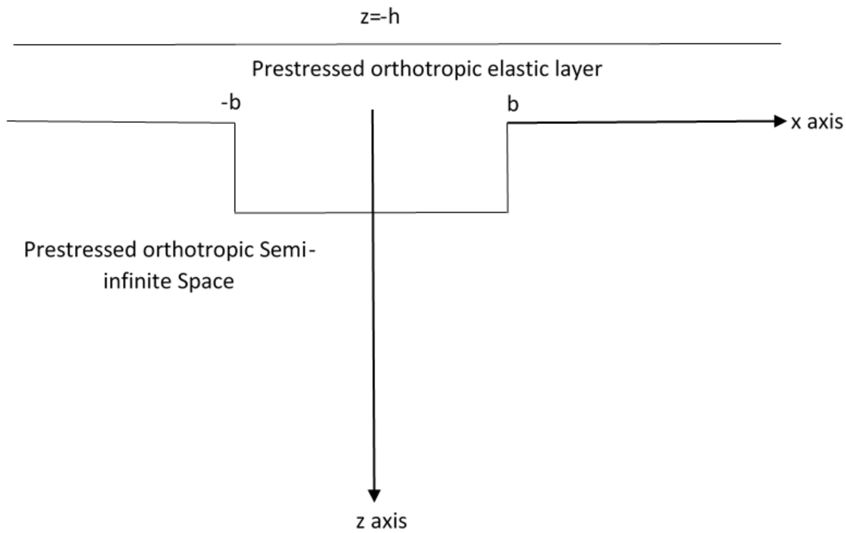

Figure 1: Geometry of Problem

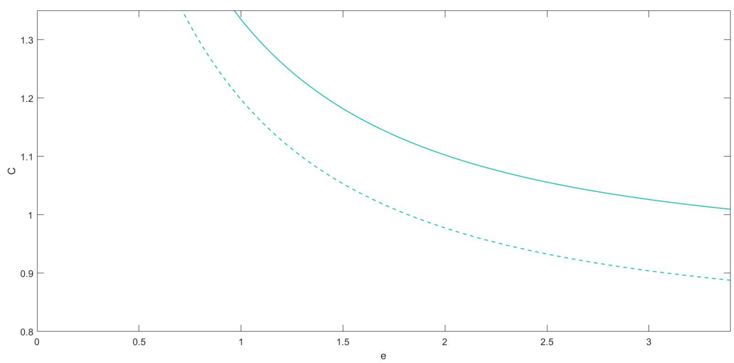

Figure 2: Variation in velocity for $P_L = 0.1356$ and $P_H = 0.1$ (denoted by solid line), $P_L = 0.3756$ and $P_H = 0.3$ (denoted by dashed-line)

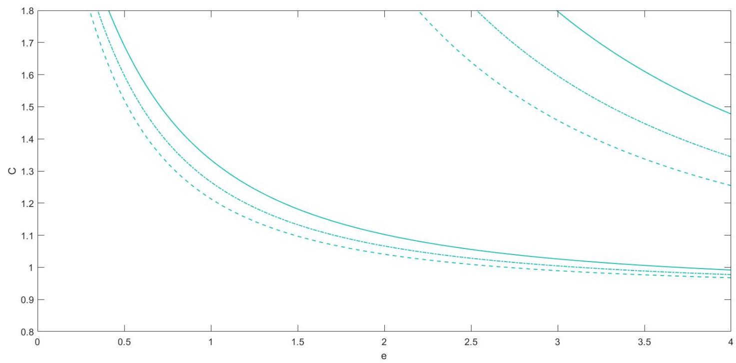

Figure 3: Variation in velocity for $P_L = 0.1356$ and $P_H = 0.1$ (denoted by solid-line), $P_L = 0.4956$ and $P_H = 0.4$ (denoted by dash-dotted line), $P_L = 0.6$ and $P_H = 0.5$ (denoted by dashed-line)

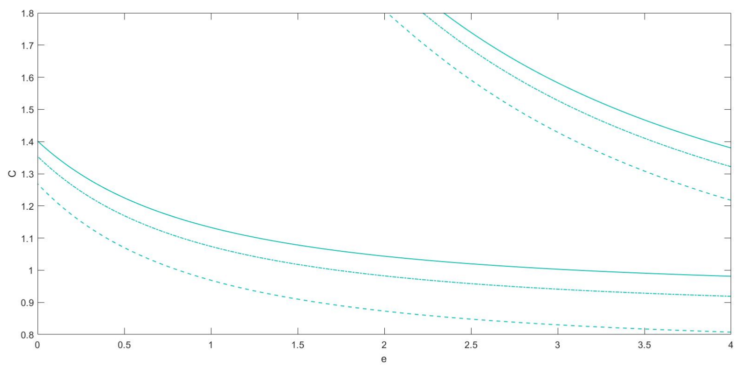

Figure 4: Variation in velocity for $P_L = 0.1356$ and $P_H = 0.1$ (denoted by solid-line), $P_L = 0.2612$ and $P_H = 0.2$ (denoted by dash-dotted line), $P_L = 0.4856$ and $P_H = 0.4$ (denoted by dashed-line)

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